EXPERIMENTAL OBSERVATION OF CONSTRUCTIVE SUPERPOSITION OF WAKE FIELDS GENERATED BY ELECTRON BUNCHES IN A DIELECTRIC-LINED WAVEGUIDE

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Abstract. We report on the experimental demonstration of a novel acceleration technique, proposed in 1999, which might deliver high acceleration gradients as required by future linear colliders. This technique utilizes constructive superposition of wake-fields produced in a dielectric-lined waveguide by short (psec) drive bunches which excite a broadband frequency spectrum having more than a hundred eigenmodes and thereby synthesize a high-amplitude accelerating field. This experiment (conducted at ATF-BNL) is compared with a related experiment by a group at the Argonne National Laboratory where the wake field consisted of a few tens of eigenmodes. We find that the axial accelerating electric field has a sharply-peaked profile with very narrow footprint as desired, and we demonstrate that fields of two bunches have been successfully superimposed.

1. INTRODUCTION

If several drive bunches are used to excite a dielectric wake field accelerator, the E_z – field at the test bunch location will have a much higher amplitude than that after only one bunch; consequently, the test bunch introduced after a train of several drive bunches will gain significantly more energy. In this acceleration scheme, all drive bunches must radiate coherently so that every consecutive drive bunch enhances the wake field produced by drive bunches which have been introduced into a DWA before it; in this case the so-called ***constructive* superposition** of wake fields occurs. Depending on the dielectric-lined waveguide parameters, and bunch charges and dimensions, an energy gain in the scale from a few hundreds MeV/m to 1 GeV/m might become available.

Constructive superposition of wake fields produced by several bunches also has been observed by a group at Argonne National Laboratory [1]. In their case, the wake field consists of a few tens of eigenmodes, and the axial E_z – field has a hill-like profile with a broad footprint (i.e. with large σ_{wake}), which relaxes requirements on the accuracy with which one must maintain the bunch spacing (or the wake field period). As a result, the E_z – field has a low peak value. In contrast, the experiment conducted at ATF generates a wake field having more than a hundred modes. The axial E_z – field has also a hill-like profile but with a narrow footprint (i.e. with small σ_{wake}), which strengthens requirements on the accuracy with which one must maintain the bunch spacing (or the wake field period). An advantage is that the E_z – field has a high peak value. Currently, forming wake fields with high peak intensity by the excitation of a large number of eigenmodes is a developing approach to achieve high accelerating gradients in DWAs [2].

2. MONITORING the ENERGY LOSSES OF CONSECUTIVE BUNCHES AS a TOOL FOR OBSERVATION OF CONSTRUCTIVE WAKE FIELD SUPERPOSITION

We use the following notation: $W(i)$ is the ith bunch energy loss per every unit of length. Q_i is the charge of the ith bunch. σ_{wake} is the coherence length (all bunches are assumed to have the same longitudinal shape). S_{12} is the spacing between bunches[a], and L is the wake field period.

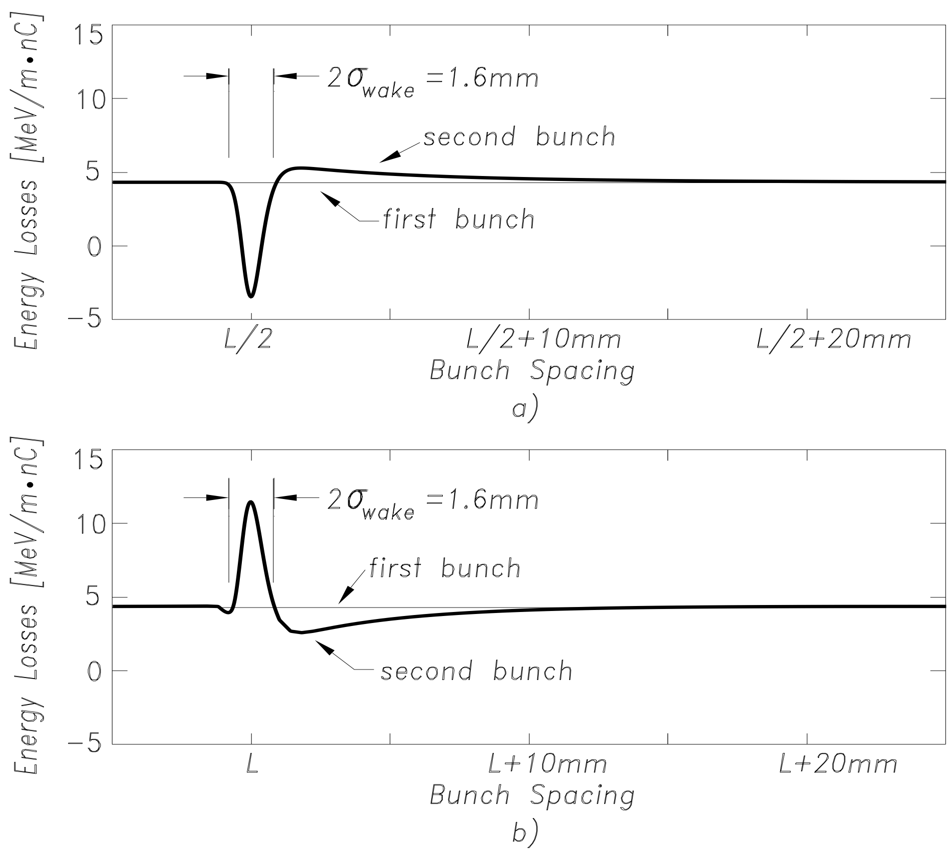

Fig. 1. The behavior of $W(1)/Q_1$ and the *resonance-like* behavior of $W(2)/Q_2$ vs. the spacing between two bunches. $W(1)/Q_1$ does not depend on the bunch spacing (see the straight, thin line). $W(2)/Q_2$ changes with the bunch spacing (see the solid, thick line). One observes that if the bunch spacing is close to $J \cdot L/2$ (where J is any integer number) the value of $W(2)/Q_2$ is significantly different from $W(1)/Q_1$. *(In this example, the coherence length σ_{wake} = 0.8mm, the wake period L=0.21022m, the structure inner radius A=1.5mm, outer radius R=19.31mm, and dielectric constant ε=9.65).*

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[a] Also can be denoted as S_b or S_{bunch}.

The behavior of $W(1)/Q_1$ and $W(2)/Q_2$ is shown in Fig. 1 (we assume that both bunches have the same rms-length $\sigma_{L2} = \sigma_{L1} = \sigma_L$.) The behavior of $W(2)/Q_2$ has a *resonance-like* character.

One discovers that if the bunch spacing is in the vicinity of $L/2 + J \cdot L$ (where J is any integer), i.e. when the second bunch weakens the wake field created by the first bunch (destructive wake field superposition), the 2nd bunch always loses less energy (per unit of charge) than the 1st bunch, i.e. if

$$L \cdot (J + 1/2) - \sigma_{wake} < S_{12} < L \cdot (J + 1/2) + \sigma_{wake} \tag{1.a}$$

then

$$\frac{W(2)}{Q_2} \leq \frac{W(1)}{Q_1} \tag{1.b}$$

As a matter of fact, the 2nd bunch can even gain energy (see negative $W(2)/Q_2$ in Fig.1.a). If $Q_2 \to 0$ and $S_{12} = L/2$, then

$$W(2) \to -2 \cdot W(1)$$

i.e. the 2nd bunch gains the energy almost twice as much as the 1st one loses.

If the bunch spacing is in the vicinity of $J \cdot L$ (where J is any integer), i.e. when the second bunch enhances the wake field created by the first bunch (constructive wake field superposition), the 2nd bunch (almost) always loses more energy (per unit of charge) than the 1st bunch, i.e. if

$$L \cdot J - \sigma_{wake} < S_{12} < L \cdot J + \sigma_{wake} \tag{2.a}$$

then

$$\frac{W(2)}{Q_2} \geq \frac{W(1)}{Q_1} \tag{2.b}$$

One finds that if every subsequent bunch **enhances** the wake field produced by the 1st bunch (constructive wake field superposition), then the distance between the i^{th} bunch and the 1st bunch must be

$$L \cdot J - \sigma_{wake} < S_{1i} < L \cdot J + \sigma_{wake} \tag{3.a}$$

where J and i are not necessarily the same, and energy losses are

$$\frac{W(N)}{Q_N} \geq \frac{W(N-1)}{Q_{N-1}} \geq \ldots \geq \frac{W(2)}{Q_2} \geq \frac{W(1)}{Q_1} \tag{3.b}$$

where N is the total number of bunches in the drive train, and it is assumed that they all have the same rms-length $\sigma_N = \sigma_{N-1} = \ldots = \sigma_2 = \sigma_1$.

If the bunch spacing is neither close to $L/2 + J \cdot L$ nor to $J \cdot L$, i.e. when the second and first bunches radiate incoherently (non-constructive superposition of wake fields), one discovers that both bunches lose almost the same energy (per unit of charge), i.e. if

$$L(J+1/2) + 12\sigma_{wake} < S_{12} < L(J+1) - \sigma_{wake} \qquad (4.a)$$

then

$$\frac{W(2)}{Q_2} \approx \frac{W(1)}{Q_1} \qquad (4.b)$$

One finds that if every subsequent bunch radiates incoherently (non-constructive superposition of wake fields), the energy losses are

$$\frac{W(N)}{Q_N} \approx \frac{W(N-1)}{Q_{N-1}} \approx \ldots \approx \frac{W(2)}{Q_2} \approx \frac{W(1)}{Q_1} \qquad (5)$$

where it is assumed that all bunches have the same rms-length $\sigma_N = \sigma_{N-1} = \ldots = \sigma_2 = \sigma_1$.

Thus, comparing Eq.(1.b), Eq.(2.b), Eq.(3.b), and Eq.(5) one concludes that monitoring the energy losses of consecutive bunches is an efficient tool to predict whether constructive wake field superposition occurs.

In the experiment conducted at ATF, we observed (see Fig. 2.a and 3.a) the difference in energy losses vs. the bunch spacing with the bunch spacing varied in the vicinity of the wake field period: this difference must demonstrate a *resonance-like* behavior (see Fig.1).

3. CONSTRUCTIVE SUPERPOSITION OF WAKE FIELDS and FINDING the WAKE FIELD PERIOD IN the DWA USED at ATF

The bunch spacing[b] S_{bunch} =700.28*psec* + ΔS_{bunch} was varied by changing the laser pulse spacing S_{laser}= 700.28*psec* + ΔS_{laser} where ΔS_{laser} was usually changed by plus/minus several psec.

Figure 2.a presents the measured difference in energy losses (marked by bars) between the 2nd and 1st bunches, $W(2) - W(1)$, versus ΔS_{bunch} when both bunches have the same rms-length σ_L = 5.4 ± 0.2psec (tail/head ratio ≈ 1.0 ÷ 1.1). The bunch charges change as shown in Fig.2.b (measured in the experiment.) For the given case, the gun operational phase is $\varphi_{gun} \approx 59.4^0$, and the corresponding LINAC phase to minimize the energy spread is $\varphi_{linac} \approx -17.91^0$ (the gun maximum field is $E_{gun} \approx$ 100 MV/m, and the LINAC field $E_{linac} \approx$ 7.6 MV/m to achieve the final bunch energy 50 MeV). With these parameters, the relationship between ΔS_{bunch} and ΔS_{laser} is as shown in Fig.2.c. With ΔS_{laser} changing from –1.39 to +5.07psec, ΔS_{bunch} changes from –1.24 to +4.88psec.

The solid curve in Fig.2.a represents the best theoretical fit that happens if the wake period is assumed

[b] The experimental setup used at ATF limits one to chose the bunch spacing to be close to the double wake field period 700.28 psec at the frequency 2856MHz (or an integer of the wake field period).

$$L = 700.28\text{psec} + \Delta\tilde{L} = 700.28\text{psec} + 3.6\text{psec} \qquad (6)^{c}$$

where $\Delta\tilde{L}$ is determined with an accuracy ± 10.5% (i.e. the wake field period is determined with an accuracy ± 0.38psec ≈ ± 115μm).

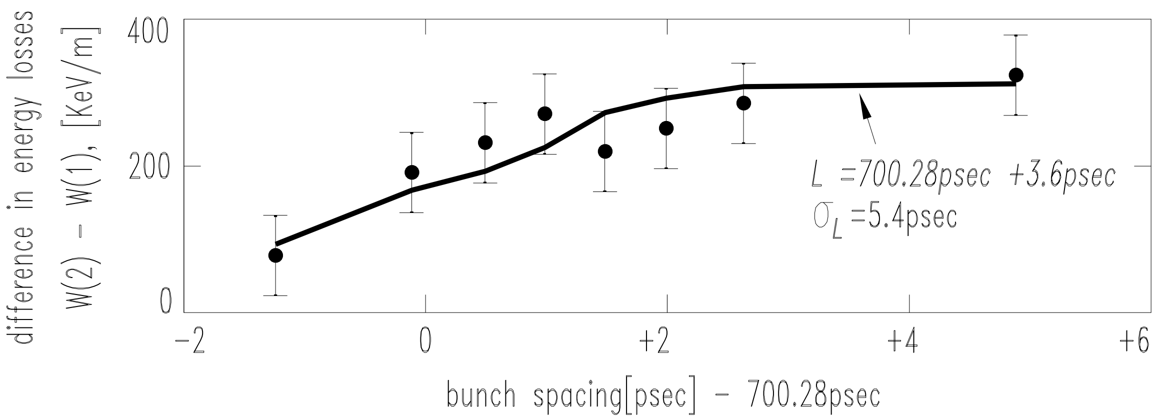

Fig. 2.a. Measured difference in energy losses (marked by bars) between the 2nd and 1st bunches, *W*(2) – *W*(1), vs. the bunch spacing. Both bunches have the same rms-length σ_L = 5.4 ± 0.2psec (tail/head ratio ≈ 1.0 ÷ 1.1). The bunch charges are shown in Fig.2.b. Along the horizontal axis $\Delta S_{bunch} = S_{bunch} - 700.28\,psec$ **is plotted, with** S_{bunch} **being the bunch spacing and 700.28psec (20.994cm) being the double RF-period. The bunch spacing** S_{bunch} **is changed by changing the laser pulse spacing** S_{laser} **as shown in Fig.2.c. The solid line represents the best theoretical fit that occurs if the wake period is assumed *L* = 700.28psec + 3.6psec (or, equivalently, *L* = 20.994cm + 1080μm).**

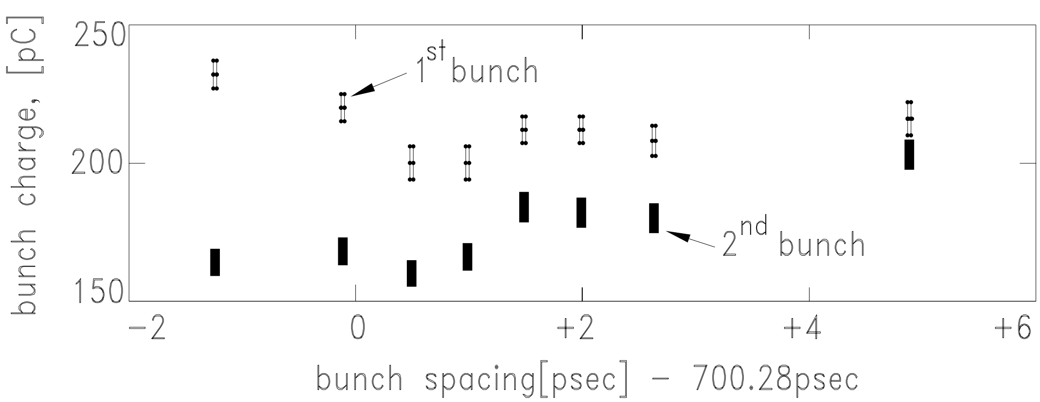

Fig. 2.b. Charges of the 2nd and 1st bunches (the accuracy ≈ ±3%). The corresponding difference in bunch energy losses, *W*(2) – *W*(1), is shown in Fig.2.a.

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[c] Or, equivalently, *L* = 20.994cm + 1080μm, where 20.994cm is the double RF-period at 2856 MHz (the speed of light c = 2.99792458·10^{8} m/sec)

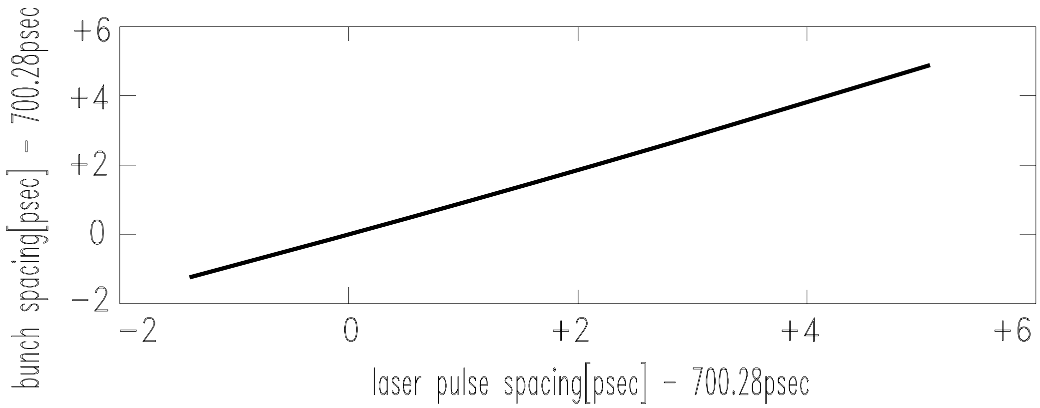

Fig. 2.c. Bunch spacing vs. the laser spacing. Along the vertical axis $\Delta S_{bunch} = S_{bunch} - 700.28 psec$ **is plotted, with** S_{bunch} **being the bunch spacing and 700.28psec (20.994cm) being the double RF-period. Along the horizontal axis** $\Delta S_{laser} = S_{laser} - 700.28 psec$ **is plotted, with** S_{laser} **being the laser pulse spacing. In this case,** $\varphi_{gun} \approx 59.4^0$, $\varphi_{linac} \approx -17.91^0$ **(to minimize the energy spread),** $E_{gun} \approx 100$ **MV/m, and** $E_{linac} \approx 7.6$ **MV/m (to achieve the final bunch energy 50 MeV). The corresponding difference in bunch energy losses,** $W(2) - W(1)$**, is shown in Fig.2.a.**

Figure 3.a presents another example.

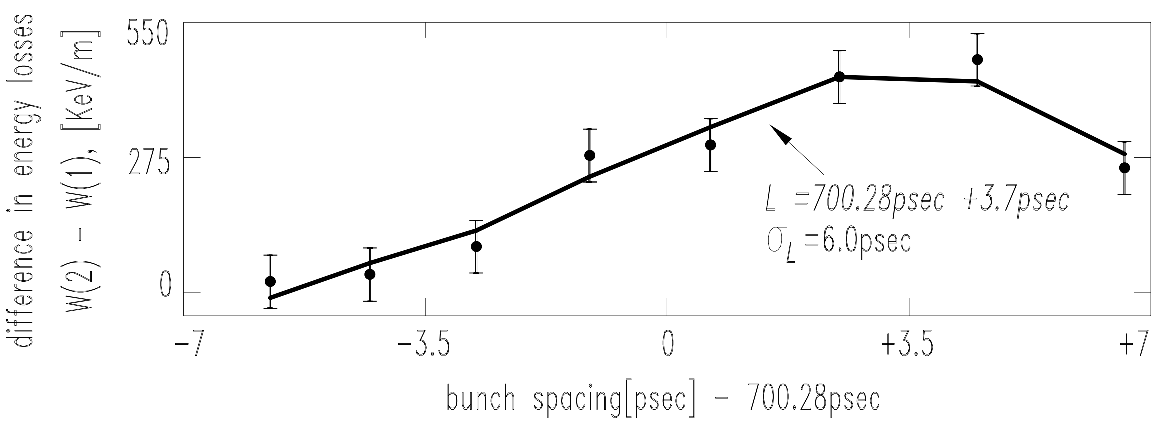

Fig. 3.a. Measured difference in energy losses (marked by bars) between the 2nd and 1st bunches, $W(2) - W(1)$**, vs. the bunch spacing. Both bunches have the same rms-length** $\sigma_L = 6.0 \pm 0.43$**psec (tail/head ratio** $\approx 1.2 \div 1.3$**). The bunch charges are shown in Fig.3.b. Along the horizontal axis** $\Delta S_{bunch} = S_{bunch} - 700.28 psec$ **is plotted, with** S_{bunch} **being the bunch spacing and 700.28psec (20.994cm) being the double RF-period. The bunch spacing** S_{bunch} **is changed by changing the laser pulse spacing** S_{laser} **as shown in Fig.3.c. The solid line represents the best theoretical fit that occurs if the wake period is assumed** $L = 700.28$**psec+ 3.7psec (or, equivalently,** $L = 20.994$**cm + 1110μm).**

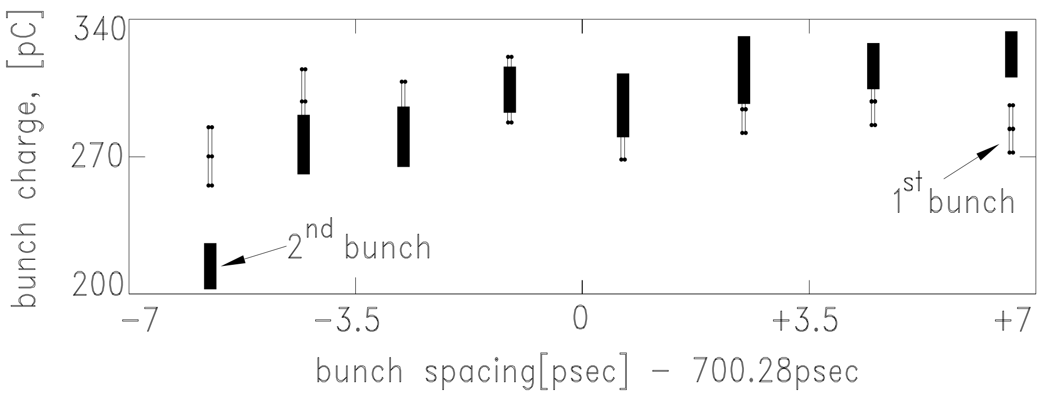

Fig. 3.b. Charges of the 2nd and 1st bunches (the accuracy ≈ ±5.5%). The difference in bunch energy losses, *W*(2) – *W*(1), is shown in Fig.3.a.

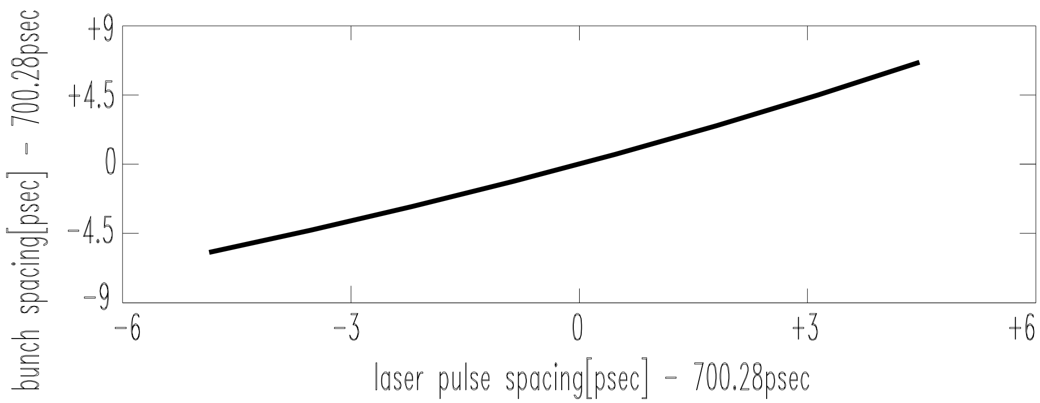

Fig. 3.c. Bunch spacing vs. the laser spacing. Along the vertical axis $\Delta S_{bunch} = S_{bunch} - 700.28psec$ **is plotted, with** S_{bunch} **being the bunch spacing and 700.28psec (20.994cm) being the double RF-period. Along the horizontal axis** $\Delta S_{laser} = S_{laser} - 700.28psec$ **is plotted, with** S_{laser} **being the laser pulse spacing. In this case,** $\varphi_{gun} \approx 57^0$**,** $\varphi_{linac} \approx -88.45^0$ **(to minimize the energy spread),** $E_{gun} \approx 51$ **MV/m, and** $E_{linac} \approx 7.94$ **MV/m (to achieve the final bunch energy 50 MeV). The corresponding difference in bunch energy losses, *W*(2) – *W*(1), is shown in Fig.3.a.**

The measured difference $W(2) - W(1)$ vs. ΔS_{bunch} is given for bunches that have $\sigma_L = 6.0 \pm 0.43$psec (tail/head ratio ≈ 1.2 ÷1.3). The bunch charges change as shown in Fig.3.b (measured in the experiment.) In this case[d], $\varphi_{gun} \approx 57^0$, $\varphi_{linac} \approx -88.45^0$, $E_{gun} \approx 51$ MV/m, and $E_{linac} \approx 7.94$ MV/m. With these parameters, the relationship between ΔS_{bunch} and ΔS_{laser} is as shown in Fig.3.c. With ΔS_{laser} changing from – 4.87 to +4.47psec, ΔS_{bunch} changes from – 5.75 to +6.62psec.

The solid curve in Fig.3.a represents the best theoretical fit that happens if the wake period is assumed

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[d] The parameters of a new gun which was installed in the summer of 2004 are given; during the DWA experiment this gun was still in a conditioning stage, and an achievable E_{gun} was low.

$$L = 700.28\text{psec} + \Delta\tilde{L} = 700.28\text{psec} + 3.7\text{psec} \qquad (7)$$

where $\Delta\tilde{L}$ is determined with an accuracy ±10.2 % (i.e. the wake field period is determined with an accuracy ± 0.38psec ≈ ± 115μm).
Eq.(7) agrees with Eq.(6) within the measurement accuracy.

The data presented in Fig.2 and 3 are fully understood by the theory, and thus, demonstrate that constructive superposition of wake fields occurs as expected.

From the frequency measurement [3], one finds that the wake period for the DWA at ATF is

$$L = 700.28\,p\sec + \Delta L = 700.28\,p\sec + 4.08\,p\sec \qquad (8)$$

where ΔL is determined with an accuracy[e] ±10%.

Eq.(6), (7), and (8) agree within the measurement accuracy, and establish the wake field period to be

$$L = 700.28\text{psec} + \Delta\tilde{L} = 700.28\text{psec} + 3.8\text{psec} \qquad (9)^{f}$$

with an accuracy of ± 0.56psec ≈ ± 170μm. It is found that to achieve constructive superposition of wake fields in a DWA similar to that at ATF one must maintain the wake field period with an accuracy better than ± 0.7psec ≈ ± 200μm. Thus, the wake field period is established with sufficient accuracy.

4. SUMMARY and DISCUSSION

We have reported on the experimental demonstration of a novel acceleration technique which was proposed by T.C. Marshall and collaborators [4], and which utilizes constructive superposition[g] of wake-fields produced by drive bunches where the drive bunches are short and excite a broadband frequency spectrum to synthesize a high-amplitude accelerating field.

Our scanning technique is different from that of the Argonne group. In their case, the measured energy spectrum of drive bunches was compared with a computed energy spectrum (see Fig. 4), while the bunch spacing was fixed and equal (within a certain accuracy) to the wake field period (i.e. $S_{bunch} = L$).

In the experiment conducted at ATF, we observed (see Fig. 2.a and 3.a) the difference in energy losses vs. the bunch spacing with the bunch spacing varied in the vicinity of the wake field period: this revealed a *resonance-like* behavior (see, e.g., Fig. 1). This observational technique has two important advantages: **a)** the wake field period can be established with excellent accuracy; **b)** agreement between theory and experiment

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[e] Assuming that the eigenfrequencies were measured with an accuracy better than ±10%.

[f] Or, equivalently, L = 20.994cm + 1140μm

[g] I.e. when the first drive bunch creates the wake-field, and the consecutive drive bunches amplify it

can be verified when the bunch spacing is different from the wake field period (i.e. $S_{bunch} \neq L$).

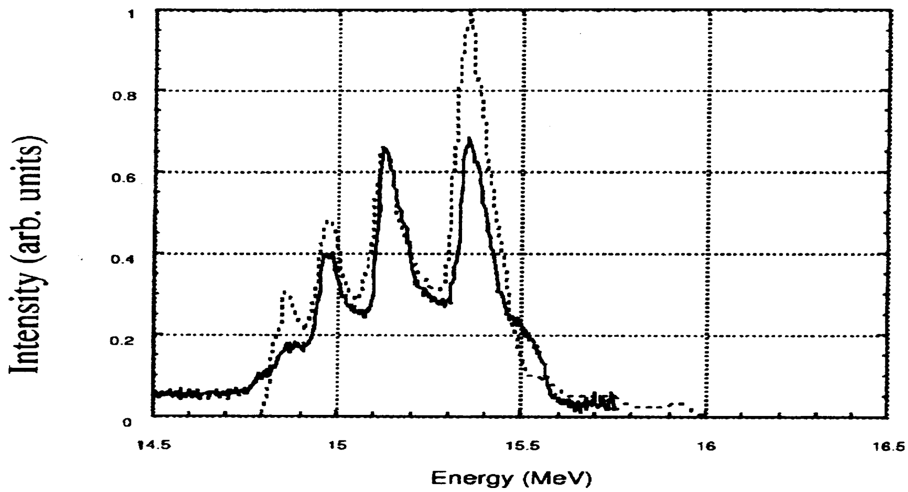

FIG.4. Energy spectrum of the 4×4.8 nC bunch train [see Fig.8. on p. 101302-6 in [1]). The solid line is the measured energy spectrum and the dotted line is the analytic fit. According to ref.8, "the overall normalization of the individual energy peaks… is not related to the magnitude of the measured wake field…" Consequently, one should compare only the peak positions along the energy (horizontal) axis but not the peak amplitudes.

A DWA has a finite energy acceptance if there is a residual dispersion at the DWA location. As a result, the bunch spacing cannot differ much from an integer of the period of RF driving the electron gun and LINAC because then the bunches would have an energy difference larger than the energy acceptance. For instance, with a typical residual dispersion as in the ATF experiment, the drive bunch spacing S_{bunch} cannot differ from an even integer of the RF-period $2J\cdot\lambda_{RF}$ by more than 7psec (for example, see Fig.3.a), i.e.

$$| S_{bunch} - 2J\cdot\lambda_{RF} | \leq 7\text{psec} \qquad (10.a)$$

where J is any integer and λ_{RF} is the RF-period.
If the wake field period is different from $2J\cdot\lambda_{RF}$, only a limited number of drive bunches whose wake fields add up constructively can be transmitted through a DWA. For the ATF experiment, constructive superposition occurs if

$$S_{bunch} = J\cdot L = J\cdot(2\lambda_{RF} + 3.8\text{psec}) \qquad (10.b)$$

where we use Eq.(9). Combining Eq.(10.a) and (10.b) one obtains that the total number of drive bunches N cannot exceed

$$N \leq J + 1 \leq \frac{7p\sec}{3.8p\sec} + 1 \approx 3 \qquad (10.c)$$

i.e. the initial energy of the 4^{th} bunch will be beyond the energy acceptance of a DWA. The acceptance of the 3^{rd} bunch is **marginal**. Consequently, the ATF experiment on constructive wake field superposition was conducted only with two bunches.

One way to overcome this limitation is to further reduce the dispersion at the DWA location. Then, the condition of Eq.(10.a) relaxes and a larger number of drive bunches whose wake fields add up constructively can be transmitted through a DWA. If, for instance, the dispersion is reduced so that Eq.(10.a) becomes $| S_{bunch} - 2J\cdot\lambda_{RF} | \leq$ 70psec, one will have $N \leq J + 1 \leq \frac{70\,p\sec}{3.8\,p\sec} + 1 \approx 20$. Regarding to the ATF experiment, the best DWA location is in the H-line where the dispersion is about 10 times or more less than in the second experimental line[h].

Adjusting the bunch spacing S_{bunch} so that it becomes equal to the wake period L (or an integer of the wake period) is suitable only if the dispersion at the DWA location is low. It is also suitable for a DWA whose every stage has the same wake field period, i.e. $L_1 = L_2 = \ldots = L_K$.

A more universal approach to overcome the limitation on the number of drive bunches is to maintain the bunch spacing equal to the double RF-period[i], $2\cdot\lambda_{RF}$, but adjust the wake period, L, so that

$$L = 2\cdot\lambda_{RF}.$$

Then, the number of drive bunches N can be any.

Finding a reliable and straightforward way to tune the wake field period in DWAs together with research on the breakdown thresholds of ceramics[j] are two major challenges that the DWA community must overcome before the practical realization of the DWA can occur.

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[h] Estimated by the author

[i] Or any even integer of the RF-period

[j] This issue is not explored in the scope of this work